

B.Sc. (Hons.) Semester - V

Subject - Mathematics

Paper - DSEMATH 501 B

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Group - A (2 x 10 = 20)

Each part of questions carries 2 marks

1. Choose the correct answers

a) The value of $e^{2\pi ni}$ is

(i) -1 (ii) 1 (iii) 0 (iv) 2

b) Expansion of $\cosh x$ in x (i) $x + \frac{x^3}{13} + \frac{x^5}{15} + \dots \infty$ (ii) $x - \frac{x^3}{13} + \frac{x^5}{15} - \dots \infty$ (iii) $1 + \frac{x^2}{12} + \frac{x^4}{14} + \dots \infty$

(iv) None of these

c) Value of $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots \infty$ (i) $\frac{\pi}{4}$ (ii) $\frac{\pi}{6}$ (iii) $\frac{\pi}{8}$ (iv) $\frac{\pi^2}{6}$ d) $(\sin \theta + i \cos \theta)^n$ is equal to(i) $\sin n\theta + i \cos n\theta$ (ii) $\cos n\theta + i \sin n\theta$ (iii) $\cos\left(\frac{n\pi}{2} - n\theta\right) + i \sin\left(\frac{n\pi}{2} - n\theta\right)$

(iv) None of these

e) $2i \sin x$ is equal to(i) $e^{ix} - e^{-ix}$ (ii) $e^x + e^{-x}$ (iii) $e^{ix} + e^{-ix}$ (iv) $e^x - e^{-x}$

- ⑧ $\vec{a} \times (\vec{b} \times \vec{c})$ is equal to
 (i) $(\vec{a} \cdot \vec{b}) \vec{c} - (\vec{a} \cdot \vec{c}) \vec{b}$ (ii) $(\vec{a} \cdot \vec{b}) \vec{a} - (\vec{a} \cdot \vec{c}) \vec{c}$
 (iii) $(\vec{a} \cdot \vec{c}) \vec{b} - (\vec{a} \cdot \vec{b}) \vec{c}$ (iv) $(\vec{a} \cdot \vec{b}) \vec{c} - (\vec{a} \cdot \vec{c}) \vec{b}$

- ⑨ If $\vec{r} = a \cos t \vec{i} + b \sin t \vec{j} + t \vec{k}$, then
 $\left| \frac{d^2 \vec{r}}{dt^2} \right|$ is equal to
 (i) a (ii) $\sqrt{a^2 + t^2}$ (iii) a + t (iv) None of these

- ⑩ Write the value of $\text{div}(\nabla \phi \times \nabla \psi)$
 (i) 1 (ii) 2 (iii) 4 (iv) 0

- ⑪ The value of $\text{Curl grad } \phi$ is
 (i) 1 (ii) -1 (iii) None of these (iv) $\vec{0}$

- ⑫ If vector function $\vec{u}(t)$ has constant direction then
 (i) $\frac{d\vec{u}}{dt} = 0$ (ii) $\frac{d\vec{u}}{dt} = \vec{0}$
 (iii) $\vec{u} \cdot \frac{d\vec{u}}{dt} = 0$ (iv) $\vec{u} \times \frac{d\vec{u}}{dt} = 0$

Group - B (5x4=20)

Each parts of questions carries 5 marks
 Answer any four question

- Apply De Moivre's theorem to solve the equation $x^9 - x^5 + x^4 - 1 = 0$
- If $\tan(\theta + i\phi) = \tan \alpha + i \sec \alpha$
 Prove that $2\theta = n\pi + \frac{\pi}{2} + \alpha$
- Find the exponential value of sine and cosine
- Show that $(1 - \frac{1}{2^2})(1 - \frac{1}{3^2})(1 - \frac{1}{5^2})(1 - \frac{1}{7^2}) \dots$
 $= \frac{6}{\pi^2}$

6. Prove that $[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}] = 2[\vec{a} \vec{b} \vec{c}]$ (3)

7. If $\vec{r} = \vec{a} \cos \omega t + \vec{b} \sin \omega t$ show that

(i) $\vec{r} \times \frac{d\vec{r}}{dt} = \omega \vec{a} \times \vec{b}$

(ii) $\frac{d^2\vec{r}}{dt^2} = -\omega^2 \vec{r}$

8. Prove that $\text{Div}(\phi \vec{a}) = \phi \text{div} \vec{a} + \vec{a} \cdot (\text{grad} \phi)$

9. Prove that the curl of the gradient of a differentiable function ϕ is a null vector.

Group - C (2x15=30)

Each question carries 15 marks

Answer any two question

10. (a) State and prove De Moivre's theorem.

(b) Find the sum of sines of n angles in A.P.

11. (a) If $u = \cos x - \frac{1}{3} \cos 3x + \frac{1}{5} \cos 5x - \dots$

$$v = \sin x - \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x - \dots$$

then P.T $\cos h(2v) = \sec x$ and

$$u = \frac{\pi}{4}$$

(b) Prove that $\tan \left\{ i \log \frac{a-ib}{a+ib} \right\} = \frac{2ab}{a^2-b^2}$

12. (a) Show that the necessary and sufficient condition for the vector function \vec{v} of the scalar variable to have constant magnitude is $\vec{v} \cdot \frac{d\vec{v}}{dt} = 0$.

⑥ If \vec{a} is a constant vector then show that $\text{curl} [\vec{r} \times (\vec{a} \times \vec{r})] = 3(\vec{r} \times \vec{a})$

13. ① Define scalar triple product and vector triple product, then

$$\text{P.T. } \vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = 0$$

② Prove that curl of velocity of a rigid body is twice its angular velocity

X

Answers of objectives

a - (ii)

b - (iii)

c - (iv)

d - (iii)

e - (i)

f - (iii)

g - (i)

h - (iv)

i - (iv)

j - (iv)